CHAPTER III

EQUATION OF STATE

3.1 DENSITY OF SEA WATER

3.1.1 SEA WATER

Seawater has been the source of life. It is where the first living and breathing organisms set fins on planet Earth. Most of the Earth's surface, approximately 70%, is covered with seawater. Scientists believed the Earth has been covered by water since shortly after the beginning of its existence.

Two of the most important variables in seawater are temperature and salinity (the concentration of dissolved salts). The two quantities work in conjunction to control the density of seawater. Since the composition of seawater is affected mainly by the addition of dissolved salts brought to it by the rivers, volcanic eruptions, erosion of rocks, and many other ways, the composition differs from one region to the next.

The density of seawater ranges from 1020 to 1030 kg/m^3 while the density of freshwater is about 1000 kg/m^3. Variations in salinity also cause the freezing point of seawater to be somewhat lower than that of freshwater. (Freshwater freezes at zero degrees Celsius.) Since salt ions interfere with the formation of hydrogen bonds, seawater does not have a fixed freezing point.

The density of seawater varies with temperature and salinity of the water. As temperature increases, density decreases. As salinity of the water increases, density also increases. Although the density of seawater varies at different points in the ocean, a good estimate of its density at the ocean's surface is 1025 kg/m^3.

3.1.2 SALINITY

Salinity is the saltiness or dissolved salt content of a body of water. It is a general term used to describe the levels of different salts such as sodium chloride, magnesium and calcium sulfates, and bicarbonates.

The technical term for saltiness in the ocean is salinity. In oceanography, it has been traditional to express salinity not as percent, but as parts per thousand (‰), which is approximately grams of salt per kilogram of solution. Other disciplines use chemical analyses of solutions, and thus salinity is frequently reported in mg/L or ppm (parts per million). Prior to 1978, salinity or halinity was expressed as ‰ usually based on the electrical conductivity ratio of the sample to "Copenhagen water", artificial sea water manufactured to serve as a world "standard". In 1978, oceanographers redefined salinity in the Practical Salinity Scale (PSS) as the conductivity ratio of a sea water sample to a standard KCl solution. Ratios have no units, so it is not the case that a salinity of 35 exactly equals 35 grams of salt per litre of solution.

3.1.3 DENSITY OF STANDARD SEA WATER

The density of standard sea water (i.e. at 1 atm), denoted ρ, is given by (Millero and Poisson, 1981)

$$\rho = \rho_0 + AS + BS^{1.5} + CS^2 \tag{3.1}$$

where ρ_0 is the density of pure water (i.e. no salinity), *S* is the salinity of sea water in ppt (parts per thousand by volume) and the coefficients *A*, *B* and *C* are functions of temperature. In the above equation,

$$\begin{aligned}\rho_0 = {} & 999.842594 + 6.793952\times10^{-2}T - 9.095290\times10^{-3}T^2 + 1.001685\times10^{-4}T^3 \\ & -1.120083\times10^{-6}T^4 + 6.536332\times10^{-9}T^5\end{aligned} \tag{3.2}$$

And according to the International one atmosphere equation (Deep-sea Research, vol. 28A, no. 6, pp 625- 629)) the coefficient of *A*, *B* and *C* are given by

$$A = 8.24493\times10^{-1} - 4.0899\times10^{-3}T + 7.6438\times10^{-5}T^2 - 8.2467\times10^{-7}T^3 + 5.3875\times10^{-9}T^4 \quad (3.3a)$$

$$B = -5.72466\times10^{-3} + 1.0227\times10^{-4}T - 1.6546\times10^{-6}T^2 \quad (3.3b)$$

$$C = 4.8314\times10^{-4} \quad (3.3c)$$

where T is the temperature in deg C. The standard error in density of sea water in Eq. (3.1) obtained using Eqs. (3.2) and (3.3) is 3.6×10^{-3} kg m^{-3}.

The coefficients *A*, *B* and *C*, in an earlier paper (Millero et al, 1976), were given by

$$A = 8.23997\times10^{-1} - 4.0644\times10^{-3}T + 7.6455\times10^{-5}T^2 - 8.3332\times10^{-10}T^3 + 5.4961\times10^{-12}T^4 \quad (3.4a)$$

$$B = -5.5078\times10^{-3} + 9.7598\times10^{-5}T - 1.6218\times10^{-6}T^2 \quad (3.4b)$$

$$C = 4.6106\times10^{-4} \quad (3.4c)$$

The standard error in density of sea water in Eq. (3.1) obtained using Eqs. (3.2) and (3.4) is 3.49×10^{-3} kg m^{-3}.

The coefficients *A*, *B* and *C*, in another earlier paper (Poisson et al, 1980), were given by

$$A = 8.24501\times10^{-1} - 4.0639\times10^{-3}T + 7.5719\times10^{-5}T^2 - 8.8910\times10^{-7}T^3 + 6.616\times10^{-9}T^4 \quad (3.5a)$$

$$B = -5.7728\times10^{-3} + 9.7437\times10^{-5}T - 1.3747\times10^{-6}T^2 \quad (3.5b)$$

$$C = 4.9054\times10^{-4} \quad (3.5c)$$

The standard error in density of sea water in Eq. (3.1) obtained using Eqs. (3.2) and (3.5) is 3.33×10^{-3} kg m^{-3}.

Implementation of Eq. (3.1) with Eqs. (3.2) and (3.3) is made in MATLAB function 'Seawaterdensity_International_ 1atm_Calc' which gives the sea water density according to International one atmosphere equation is given in Table 3.1a.

Table 3.1a: Seawaterdensity_International_1atm_Calc

```
%*****************************************************************
% Equation of State According to International One Atmosphere
% Equation
%*****************************************************************

function[relative_density,sea_water_density,water_density]=
Seawaterdensity_International_1atm_Calc(temperature,salinity)

A= 8.24493e-1- 4.0899e-3*temperature+ 7.6438e-5*temperature^2...
   - 8.2467e-7*temperature^3+ 5.3875e-9*temperature^4;

B= -5.72466e-3 + 1.0227e-4*temperature- 1.6546e-6*temperature^2;

C= 4.8314e-4;

%Calculating the water density
water_density= 999.842594+ 6.793952e-2*temperature-9.095290e-...
      3*temperature^2+1.001685e-4*temperature^3-1.120083e-6*...
      *temperature^4+ 6.536336e-9*temperature^5;

%Calculating the sea water density

sea_water_density= water_density+ A*salinity + B*...
                  (salinity^1.5)+C*(salinity^2);

%Calculating the relative density

relative_density= sea_water_density- water_density;


%----------------------End of Function--------------------------
```

Implementation of Eq. (3.1) with Eqs. (3.2) and (3.4) is made in MATLAB function 'Seawaterdensity_Millero_Calc' which gives the sea water density according to Millero's equation is given in Table 3.2a and implementation of Eq. (3.1) with Eqs. (3.2) and (3.5) is made in MATLAB function 'Seawaterdensity_Poisson_Calc' which gives the sea water density according to Poisson's equation is given in Table 3.3a.

Table 3.2a: Seawaterdensity_Millero_Calc

```
%*****************************************************************
%        Equation of State According to Millero (1976)
%*****************************************************************
function[relative_density,sea_water_density,water_density]= ...
        Seawaterdensity_Millero_Calc(temperature,salinity)

A= 8.23997e-1-4.0644e-3*temperature+7.6455e-5*temperature^2-...
   8.3332e-10*temperature^3+5.4961e-12*temperature^4;

B= -5.5078e-3+ 9.7598e-5*temperature- 1.6218e-6*temperature^2;

C= 4.6106e-4;

water_density= 999.842594 + 6.793952e-2*temperature-...
               9.095290e-3*temperature^2+ 1.001685e-...
               4*temperature^3- 1.120083e-6*temperature^4+...
               6.536336e-9*temperature^5;

sea_water_density= water_density + A*salinity +...
                   B*salinity^1.5+C*salinity^2;
releative_density=sea_water_density- water_density;

%-----------------------End of Function---------------------------
```

Table 3.3a: Seawaterdensity_Poisson_Calc

```
%*****************************************************************
%            Equation of State According to Poisson (1980)
%*****************************************************************
function[relative_density,sea_water_density,water_density]=...
        Seawaterdensity_Millero_Calc(temperature,salinity)
A= 8.24501e-1- 4.0639e-3*temperature+ 7.5719e-5*...
   temperature^2-8.8910e-7*temperature^3+ 6.616e-...
   9*temperature^4;

B= -5.7728e-3 + 9.7437e-5*temperature-1.3747e-6*temperature^2;

C= 4.9054e-4;

water_density= 999.842594 + 6.793952e-2*temperature- ...
               9.095290e-3*temperature^2+ 1.001685e- ...
               4*temperature^3- 1.120083e-6*temperature^4+ ...
               6.536336e-9*temperature^5;

sea_water_density= water_density+A*salinity+B*(salinity^1.5)+...
                   C*(salinity^2);
relative_density= sea_water_density- water_density;
%-----------------------End of Function-------------------------
```

Typical results obtained using this functions are given in Table 3.1b to Table 3.3b .

Table 3.1 (b): Typical computed values of sea water density using MATLAB function ‘Seawaterdensity_International_1atm_Calc’
(Values in parenthesis are from Millero and Poisson, 1980)

Temperature	Salinity				
	0	10	20	35	40
0	999.843 (999.843)	1007.950 (1007.955)	1016.01 (1016.014)	1028.11 (1028.106)	1032.15 (1032.147)
15	999.102 (999.102)	1006.78 (1006.784)	1014.44 (1014.443)	1025.97 (1025.973)	1029.83 (1029.834)
30	995.651 (995.651)	1003.10 (1003.095)	1010.53 (1010.527)	(1021.729) 1021.73	1025.48 (1025.483)
40	992.220 (992.220)	999.575 (999.575)	1006.91 (1006.915)	1017.97 (1017.973)	1021.68 (1021.679)

Table 3.2(b): Typical computed values of sea water density using MATLAB function ‘Seawaterdensity_Millero_Calc’

Temperature	Salinity				
	5	10	20	35	40
0	1003.91	1007.95	1016.01	1028.11	1032.15
15	1002.96	1006.81	1014.50	1026.06	1029.93
30	999.472	1003.28	1010.89	1022.36	1026.20
40	996.104	999.971	1007.70	1019.35	1023.24

Table 3.3(b): Typical computed values of sea water density using MATLAB function ‘Seawaterdensity_Poisson_Calc’

Temperature	Salinity				
	5	10	20	35	40
0	1003.91	1007.95	1016.01	1028.11	1032.15
15	1002.95	1006.78	1014.44	1025.97	1029.83
30	999.378	1003.09	1010.52	1021.73	1025.49
40	995.903	999.572	1006.91	1017.99	1021.70

3.1.4 DENSITY OF SEA WATER AT HIGH PRESSURE

The density of sea water at high pressure, denoted by ρ , is given by

$$\rho(S,T,P) = \frac{\rho(S,T,0)}{1 - P / k(S,T,P)} \tag{3.6}$$

where S is the salinity of sea water in ppt , T is the temperature, P is the applied pressure and $\rho(S, t, 0)$ is the density of the sea water according to one atmosphere International Equation of State, 1980.

$k(S, t, P)$ is the Secant bulk modulus given by

$$k(S,T,P) = k(S,T,0) + AP + BP^2 \tag{3.7}$$

where $k(S, t, 0)$ and the co-efficient A,B are the function of salinity and temperature, and is given by

$$\begin{aligned} k(S,T,0) = k_w &+ (57.6746 - 0.603459T + 1.09987 \times 10^{-2} T^2 - 6.1670 \times 10^{-5} T^3)S \\ &+ (7.944 \times 10^{-2} + 1.6483 \times 10^{-2} T - 5.3009 \times 10^{-4} T^2) S^{1.5} \end{aligned} \tag{3.8a}$$

$$\begin{aligned} A = A_w &+ (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5} T - 1.6078 \times 10^{-6} T^2)S \\ &+ 1.91075 \times 10^{-4} S^{1.5} \end{aligned} \tag{3.8b}$$

$$B = B_w + (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8} T + 9.1697 \times 10^{-10} T^2)S \tag{3.8c}$$

and the pure water terms k_w, A_w and B_w of Eq. (3.7) are given by

$$\begin{aligned} k_w = &19652.21 + 148.4206T - 2.327105T^2 + 1.360477 \times 10^{-2} T^3 \\ &- 5.155288 \times 10^{-5} T^4 \end{aligned} \tag{3.9a}$$

$$A_w = 3.239908 + 1.43713 \times 10^{-3} T + 1.16092 \times 10^{-4} T^2 - 5.77905 \times 10^{-7} T^3 \tag{3.9b}$$

$$B_w = 8.50935 \times 10^{-5} - 6.12293 \times 10^{-6} T + 5.2787 \times 10^{-8} T^2 \tag{3.9c}$$

Implementation of Eq. (3.6) with Eq. (3.7) ,(3.8) and (3.9) is made in MATLAB function ‘Seawaterdensity_International_highpressure_Calc’ and is given in Table 3.4(a).

Table 3.4(a) : Seawaterdensity_International_highpressure_Calc

```
%*******************************************************************
%       Equation of State of Sea Water At High Pressure
%*******************************************************************
function [releative_density,density_seawater,density_water]=
Seawaterdensity_International_highpressure_Calc(temperature,salinity,pr
essure)

t= temperature;
S= salinity;
P= pressure;

%=====================================================================
%                Calculating Secant Bulk Modulus
%=====================================================================

kw= 19652.21+ 148.4206*t- 2.327105*t^2+ 1.360477e-2*(t^3)-...
    5.155288e-5*(t^4);
Aw= 3.239908+ 1.43713e-3*t+ 1.16092e-4*t^2- 5.77905e-7*t^3;
Bw= 8.50935e-5- 6.12293e-6*t + 5.2787e-8*(t^2);
k0= kw + (54.6746- 0.603459*t+ 1.09987e-2*(t^2)- 6.1670e-...
    5*(t^3))*S +(7.944e-2 + 1.6483e-2*t- 5.3009e4*(t^2))*...
    (S^1.5);
A= Aw+ (2.2838e-3- 1.0981e-5*t- 1.6078e-6*(t^2))*S+ 1.91075e-...
   4*(S^1.5);
B= Bw+ (-9.9348e-7+ 2.0816e-8*t+ 9.1697e-10*t^2)*S;
bulk_modulus= k0+ A*P+ B*P^2;

%=====================================================================
%    One atmoSphere International Equation of State [1980]
%=====================================================================

A= 8.24493e-1- 4.0899e-3*t+ 7.6438e-5*t^2- 8.2467e-7*t^3+...
5.3875e-9*t^4;
B= -5.72466e-3 + 1.0227e-4*t- 1.6546e-6*t^2;
C= 4.8314e-4;
rho_w= 999.842594 + 6.793952e-2*t- 9.095290e-3*t^2+...
       1.001685e-4*t^3-1.120083e-6*t^4+ 6.536336e-9*t^5;
rho_zero= rho_w+ A*S + B*(S^1.5)+ C*(S^2);

%=====================================================================
%   The High Pressure International Equation of State of
%   Seawater,1980
%=====================================================================
density_seawater= rho_zero/(1- (P/bulk_modulus));

density_water= rho_w;

releative_density= density_seawater- density_water;

%-------------------------End of Function-----------------------------
```

Table 3.4(b): Typical computed values of sea water density using MATLAB function 'Seawaterdensity_International_highpressure_Calc'
(Values in parenthesis are from Millero,Poisson, Bradshaw,Schleicher (1980))

S (°/oo)	T°C	P (bars)	Sea water Density
0	5	0	999.967 (999.96675)
		1000	1044.13 (1044.12802)
	25	0	997.048 (997.04796)
		1000	1037.95 (1037.90204)
35	5	0	1027.68 (1027.67547)
		1000	1069.49 (1069.48914)
	25	0	1023.34 (1023.34306)
		1000	1062.59 (1062.53817)

3.2 SOUND SPEED IN SEA WATER

3.2.1 MACKENZIE'S EQUATION (1981)

The equation of the sound speed (*C*) in sea-water as a function of temperature (*T*), salinity (*S*) and depth (*D*), given by Mackenzie (1981) is as follows

$$\begin{aligned} C(D,S,T) &= 1448.96 + 4.591T - 5.304\times10^{-2}T^2 + 2.374\times10^{-4}T^3 + 1.340(S-35) \\ &+ 1.630\times10^{-2}D + 1.675\times10^{-7}D^2 - 1.025\times10^{-2}T(S-35) - 7.139\times10^{-13}TD^3 \end{aligned} \tag{3.10}$$

where *T* is temperature in degrees Celsius, *S* is salinity in parts per thousand and *D* is depth in meters. This equation is valid from temperature – 2 to 30 °C, salinity 25 to 40 parts per thousand, and depth 0 to 8000 m.

3.2.2 COPPENS'S EQUATION (1981)

The equation of the sound speed (*C*) in sea-water as a function of temperature (*T*), salinity (*S*) and depth (*D*), given by Coppens (1981) is as follows

$$\begin{aligned} C(D,S,T) &= C(0,S,T) + (16.23 + 0.253t)D + (0.213 - 0.1t)D^2 \\ &+ [0.016 + 0.0002(S-35)](S-35)tD \end{aligned} \tag{3.11a}$$

$$C(0,S,t) = 1449.05 + 45.7t - 5.21t^2 + 0.23t^3 + (1.333 - 0.126t + 0.009t^2)(S - 35) \quad (3.11b)$$

where $t = T/10$, T is temperature in degrees Celsius, S is salinity in parts per thousand. and D is depth in kilometers. This equation is valid from temperature –2 to 35 °C, salinity 0 to 45 parts per thousand, and depth 0 to 4000 m.

3.2.3 THE UNESCO EQUATION: CHEN AND MILLERO (1977)

This international standard algorithm, often known as the UNESCO algorithm, is due to Chen and Millero (1977), and has a more complicated form than the simple equations above, but uses pressure as a variable rather than depth. For the original UNESCO paper see Fofonoff and Millard (1983). Wong and Zhu (1995) recalculated the coefficients in this algorithm following the adoption of the International Temperature Scale of 1990 and their form of the UNESCO equation is as follows

$$C(S,T,P) = C_w(T,P) + A(T,P)S + B(T,P)S^{1.5} + D(T,P)S^2 \quad (3.12a)$$

$$\begin{aligned} C_w(T,P) = &(C_{00} + C_{01}T + C_{02}T^2 + C_{03}T^3 + C_{04}T^4 + C_{05}T^5) \\ &+ (C_{10} + C_{11}T + C_{12}T^2 + C_{13}T^3 + C_{14}T^4)P \\ &+ (C_{20} + C_{21}T + C_{22}T^2 + C_{23}T^3 + C_{24}T^4)P^2 \\ &+ (C_{30} + C_{31}T + C_{32}T^2)P^3 \end{aligned} \quad (3.12b)$$

$$\begin{aligned} A(T,P) = &(A_{00} + A_{01}T + A_{02}T^2 + A_{03}T^3 + A_{04}T^4) \\ &+ (A_{10} + A_{11}T + A_{12}T^2 + A_{13}T^3 + A_{14}T^4)P \\ &+ (A_{20} + A_{21}T + A_{22}T^2 + A_{23}T^3 + A_{24}T^4)P^2 \\ &+ (A_{30} + A_{31}T + A_{32}T^2)P^3 \end{aligned} \quad (3.12c)$$

$$B(T,P) = B_{00} + B_{01} + (B_{10} + B_{11}T)P \quad (3.12d)$$

$$D(T,P) = D_{00} + D_{10}P \quad (3.12e)$$

where T is temperature in degrees Celsius, S is salinity in parts per thousand and P is pressure in bar. The coefficients of the above equations are given by

Coefficients	Numerical Value	Coefficients	Numerical Value
C_{00}	1402.388	A_{02}	7.166×10^{-5}
C_{01}	5.03830	A_{03}	2.008×10^{-6}
C_{02}	-5.81090×10^{-2}	A_{04}	-3.21×10^{-8}
C_{03}	3.3432×10^{-4}	A_{10}	9.4742×10^{-5}
C_{04}	-1.47797×10^{-6}	A_{11}	-1.2583×10^{-5}
C_{05}	3.1419×10^{-9}	A_{12}	-6.4928×10^{-8}
C_{10}	0.153563	A_{13}	1.0515×10^{-8}
C_{11}	6.8999×10^{-4}	A_{14}	-2.0142×10^{-10}
C_{12}	-8.1829×10^{-6}	A_{20}	-3.9064×10^{-7}
C_{13}	1.3632×10^{-7}	A_{21}	9.1061×10^{-9}
C_{14}	-6.1260×10^{-10}	A_{22}	-1.6009×10^{-10}
C_{20}	3.1260×10^{-5}	A_{23}	7.994×10^{-12}
C_{21}	-1.7111×10^{-6}	A_{30}	1.100×10^{-10}
C_{22}	2.5986×10^{-8}	A_{31}	6.651×10^{-12}
C_{23}	-2.5353×10^{-10}	A_{32}	-3.391×10^{-13}
C_{30}	-9.7729×10^{-9}	B_{01}	-4.42×10^{-5}
C_{31}	3.8513×10^{-10}	B_{10}	7.3637×10^{-5}
C_{32}	-2.3654×10^{-12}	B_{11}	1.7950×10^{-7}
A_{00}	1.389	D_{00}	1.727×10^{-3}
A_{01}	-1.262×10^{-2}	D_{10}	-7.9836×10^{-6}

This equation is valid from temperature 0 to 40 °C, salinity 0 to 40 parts per thousand, and pressure 0 to 1000 bars.

3.2.4 DEL GROSSO'S EQUATION (1974)

An alternative equation to the UNESCO algorithm, which has a more restricted range of validity, but which is preferred by some authors, is the Del Grosso equation (1974). Wong and Zhu (1995) also reformulated this equation for the new 1990 International Temperature Scale and their version is:

$$C(S,T,P) = C_{000} + \Delta C_T + \Delta C_S + \Delta C_P + \Delta C_{STP} \tag{3.14a}$$

$$\Delta C_T(T) = C_{T1}T + C_{T2}T^2 + C_{T3}T^3 \tag{3.14b}$$

$$\Delta C_S(S) = C_{S1}S + C_{S2}S^2 \tag{3.14c}$$

$$\Delta C_P(P) = C_{P1}P + C_{P2}P^2 + C_{P3}P^3 \tag{3.14d}$$

$$\begin{aligned}\Delta C_{STP}(S,T,P) = {} & C_{TP}TP + C_{T3P}T^3P + C_{TP2}TP^2 + C_{T2P2}T^2P^2 + C_{TP3}TP^3 \\ & + C_{ST}ST + C_{ST2}ST^2 + C_{STP}STP + C_{S2TP}S^2TP + C_{S2P2}S^2P^2\end{aligned} \tag{3.14e}$$

where *T* is temperature in degrees celsius, *S* is salinity in parts per thousand and *P* is pressure in kg/cm^2. The coefficients of the above equations are given by

Coefficients	Numerical Value	Coefficients	Numerical Value
C_{000}	1402.392	C_{TP}	0.6353509×10^{-2}
C_{T1}	0.5012285×10^{1}	C_{T2P2}	0.2656174×10^{-7}
C_{T2}	-0.551184×10^{-1}	C_{TP2}	$-0.1593895 \times 10^{-5}$
C_{T3}	0.221649×10^{-3}	C_{TP3}	0.5222483×10^{-9}
C_{S1}	0.1329530×10^{1}	C_{T3P}	$-0.4383615 \times 10^{-6}$
C_{S2}	0.1288598×10^{-3}	C_{S2P2}	$-0.1616745 \times 10^{-8}$

Coefficients	Numerical Value	Coefficients	Numerical Value
C_{P1}	0.1560592	C_{ST2}	0.9688441×10^{-4}
C_{P2}	0.2449993×10^{-4}	C_{S2TP}	0.4857614×10^{-5}
C_{P3}	$-0.8833959 \times 10^{-8}$	C_{STP}	$-0.3406824 \times 10^{-3}$
C_{ST}	$-0.1275936 \times 10^{-1}$		

This equation is valid from temperature 0 to 30 °C, salinity 30 to 40 parts per thousand, and pressure 0 to 1000 kg/cm^2, where 100 kPa = 1.019716 kg/cm^2.

Eq. (3.10), Mackenzie's Equation, is implemented in the MATLAB function 'Sound_speed_Mackenzie_Calc' and is given in Table 3.5.

Table 3.5: Sound_speed_Mackenzie_Calc

```
%****************************************************
%        Sound Speed in Sea by Mackenzie's Equation
%****************************************************
function [sound_speed]=
Sound_speed_Mackenzie_Calc(temperature,depth,salinity)
sound_speed= 1448.96+ 4.591*temperature-5.304e-...
2*temperature^2+2.374e-4*temperature^3+1.340*...
(salinity-35)+1.025e-2*temperature*(salinity-35)-...
7.139e-13*temperature*depth^3;
%-----------------End of the Function------------------
```

Eq. (3.11), Coppen's Equation, is implemented in the MATLAB function 'Sound_speed_ Coppen_Calc' and is given in Table 3.6.

Table 3.6: Sound_speed_Coppen_Calc

```
%*****************************************************
%       Sound Speed in Sea by Coppen's Equation
%*****************************************************
function[sound_speed]=
Sound_speed_Coppen_Calc(temperature,depth,salinity)
t= temperature/10;
D= depth/1000;    %converting depth in KM
S= salinity;
c0= 1449.05+ 45.7*t- 5.21*(t^2)+ 0.23*(t^3)+ (1.333-...
    0.126*t+0.009*t^2)*(S- 35);
sound_speed= c0+ (16.23+ 0.253*t)*D+ (0.213-...
             0.1*t)*(D^2)+(0.016+ 0.0002*(S- 35))* ...
            (S- 35)*t*D;
%---------------------End of the Code--------------------
```

Eq.(3.12),UNESCO Equation, is implemented in the MATLAB function 'Sound_speed_ChenandMillero_Calc' and is given in Table 3.7.

Table 3.7: Sound_speed_ChenandMillero_Calc

```
%***********************************************************
%     Sound Speed in Sea by Chen and Millero [1977] [UNESCO]
%***********************************************************
function[sound_speed]=
sound_speed_ChenandMillero_Calc(temperature,salinity,pressure)

T= temperature;
S= salinity;
P= pressure*1e-2;
%1 bar = 100 kPa ; converting Pressure from kPa to bar.

C00= 1402.388;      A02= 7.166e-5;
C01= 5.03830;       A03= 2.008e-6;
C02=-5.81090e-2;    A04=-3.21e-8;
C03= 3.3432e-4;     A10= 9.4742e-5;
C04=-1.47797e-6;    A11=-1.2583e-5;
C05= 3.1419e-9;     A12=-6.4928e-8;
C10= 0.153563;      A13= 1.0515e-8;
C11= 6.8999e-4;     A14=-2.0142e-10;
C12=-8.1829e-6;     A20=-3.9064e-7;
C13= 1.3632e-7;     A21= 9.1061e-9;
C14=-6.1260e-10;    A22=-1.6009e-10;
C20= 3.1260e-5;     A23= 7.994e-12;
C21=-1.7111e-6;     A30= 1.100e-10;
C22= 2.5986e-8;     A31= 6.651e-12;
C23=-2.5353e-10;    A32=-3.391e-13;
C24= 1.0415e-12;    B00=-1.922e-2;
C30=-9.7729e-9;     B01=-4.42e-5;
C31= 3.8513e-10;    B10= 7.3637e-5;
C32=-2.3654e-12;    B11= 1.7950e-7;
A00= 1.389;         D00= 1.727e-3;
A01=-1.262e-2;      D10=-7.9836e-6;

A = (A00 + A01*T + A02*T^2 + A03*T^3 + A04*T^4)+...
    (A10 + A11*T + A12*T^2 + A13*T^3 + A14*T^4)*P +...
    (A20 + A21*T + A22*T^2 + A23*T^3)*P^2 +...
    (A30 + A31*T + A32*T^2)*P^3;

B =  B00 + B01*T + (B10 + B11*T)*P;

C = (C00 + C01*T + C02*T^2 + C03*T^3 + C04*T^4 + C05*T^5)+...
    (C10 + C11*T + C12*T^2 + C13*T^3 + C14*T^4)*P +...
    (C20 +C21*T +C22*T^2 + C23*T^3 + C24*T^4)*P^2 +...
    (C30 + C31*T + C32*T^2)*P^3;

D = D00 + D10*P ;

sound_speed= C+ A*S+ B*(S^1.5)+ D*(S^2);
%------------------End of the Function---------------------------
```

Eq. (3.14), Del Grosso's equation, is implemented in the MATLAB function 'Sound_speed_Del_Grosso_Calc' and is given in Table 3.8.

Table 3.8: Sound_speed_Del_Grosso_Calc

```
%****************************************************************
%                 Sound Speed Calculated by Del Grosso
%****************************************************************

function[sound_speed]=
sound_speed_Del_Grosso_Calc(temperature,salinity,pressure)

T= temperature;
S= salinity;
P= pressure;

C000=  1402.392;
Ct1=   0.5012285e1;
Ct2=  -0.551184e-1;
Ct3=   0.221649e-3;
Cs1=   0.1329530e1;
Cs2=   0.1288598e-3;
Cp1=   0.1560592;
Cp2=   0.2449993e-4;
Cp3=  -0.8833959e-8;
Cst=  -0.1275936e-1;
Ctp=   0.6353509e-2;
Ct2p2= 0.2656174e-7;
Ctp2= -0.1593895e-5;
Ctp3=  0.5222483e-9;
Ct3p= -0.4383615e-6;
Cs2p2=-0.1616745e-8;
Cst2=  0.9688441e-4;
Cs2tp= 0.4857614e-5;
Cstp= -0.3406824e-3;

Ct = Ct1*T + Ct2*T^2 + Ct3*T^3;

Cs = Cs1*S + Cs2*S^2;

Cp = Cp1*P + Cp2*P^2 + Cp3*P^3;

Cstp = Ctp*T*P + Ct3p*T^3*P + Ctp2*T*P^2 + Ct2p2*T^2*P^2 + ...
       Ctp3*T*P^3 + Cst*S*T + Cst2*S*T^2 + Cstp*S*T*P + ...
       Cs2tp*S^2*T*P + Cs2p2*S^2*P^2;

sound_speed = C000 + Ct + Cs + Cp + Cstp ;

%----------------------End of Function---------------------------
```

3.3 CONVERSION OF PRESSURE AND DEPTH

Both the UNESCO equation and Del Grosso's equation use pressure as a variable instead of depth because they are based on measurements made in small laboratory pressurized chambers. Useful guidance and suitable equations for converting pressure into depth and depth into pressure can be found in Leroy and Parthiot (1998). The key equations here are:

3.2.1 CONVERSION OF PRESSURE INTO DEPTH

The depth (*Z*) can be expressed as a function of Pressure, *P*, and the latitude of the place, θ .The expression is given by

$$Z(P,\theta) = 9.72659\times10^{2}P - 2.2512\times10^{-1}P^{2} + 2.279\times10^{-4}\mathrm{P}^{3} - 1.82\times10^{-7}\mathrm{P}^{4} + 1.092\times10^{-4}P + g(\theta) \tag{3.15a}$$

where $g(\theta)$, the international formula for gravity, is given by,

$$g(\theta) = 9.780318(1 + 5.2788\times10^{-3}\sin^{2}\theta + 2.36\times10^{-5}\sin^{4}\theta) \tag{3.15b}$$

where depth in meters, pressure in MPa (relative to atmospheric pressure).

3.2.2 CONVERSION OF DEPTH INTO PRESSURE

The Pressure (*P*) can be expressed as a function of depth, Z, and the latitude of the place, θ .The expression is given by

$$P(Z,\theta) = h(Z,\theta) \tag{3.16a}$$

$$h(Z,\theta) = h(Z,45)\times k(Z,\theta) \tag{3.16b}$$

$$h(Z,45) = 1.00818\times10^{-2}Z + 2.465\times10^{-8}Z^{2} - 1.25\times10^{-13}Z^{3} + 2.8\times10^{-19}Z^{4} \tag{3.16c}$$

$$k(Z,\theta) = \frac{g(\theta) - 2\times10^{-5}Z}{9.80612 - 2\times10^{-5}Z} \tag{3.16d}$$

$$g(\theta) = 9.780318(1 + 5.2788 \times 10^{-3} \sin^2 \theta + 2.36 \times 10^{-5} \sin^4 \theta) \qquad (3.16e)$$

where depth in metres and pressure in MPa (relative to atmospheric pressure).

Implementation of Eq. (3.15) and (3.16) ,conversion of depth and pressure,is made in MATLAB function ‘Pressure_and_Depth_Calc’and is given in the Table 3.9.

Table 3.9: Pressure_and_Depth_Calc

```
%**********************************************************
%               Pressure and Depth Conversion
%**********************************************************
function[depth,pressure]=...
Pressure_and_Depth_Calc(latitude,pressure,depth,option)

%optiton =1 ---> Pressure to Depth
%optiton =2 ---> Depth to Pressure

if option == 1
   theta = (latitude)*(pi/180);
   %latitude in deg
   P = (pressure)/1000;
   %pressure in MPa (relative to atmospheric pressure)
   g= 9.780318*(1 + 5.2788e-3*((sin(theta))^2)+ 2.36e-...
      5*((sin(theta))^4));
   Z= P*9.72659e2-(P^2)*2.2512e-1+(P^3)*2.279e-4-(P^4)*g*...
      1.82e-7+P*1.092e-4;
   depth= Z/10;
end

if option == 2
   theta = latitude*(pi/180);
   %latitude in deg
   Z = depth;
   %depth in metres
   g= 9.780318*(1 + 5.2788e-3*((sin(theta))^2)+ 2.36e-...
      5*((sin(theta))^4));
   k =(g - Z*2e-5)/(9.80612 - Z*2e-5);
   h_45= Z*1.00818e-2+ (Z^2)*2.465e-8- (Z^3)*1.25e-13+ ...
         (Z^4)*2.8e-19;
   h_theta= h_45*k;
   pressure= h_theta*1000;
end
%--------------------End of Function-----------------------
```